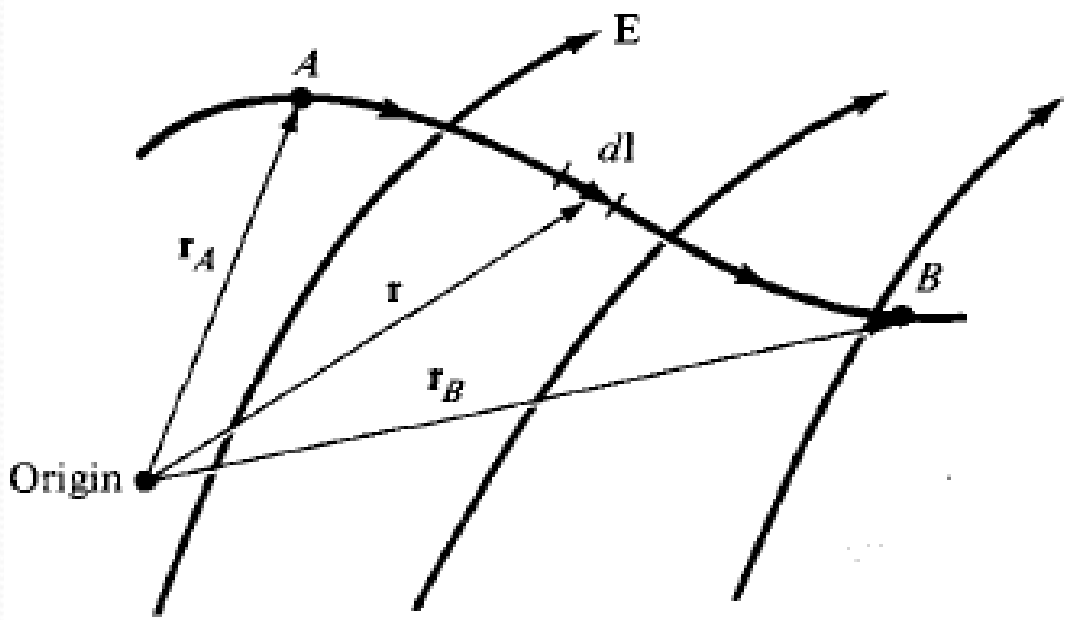
LECTURE NO 17

Electrostatics

TOPIC COVERED

- DEFINE ELECTRIC POTENTIAL
- EXPRESSION FOR ELECTRIC POTENTIAL


E
A
dl
$\mathbf{r}_A$
r
B
$\mathbf{r}_B$
Origin

Suppose we wish to move a point charge Q from point A to point B in an electric field $\mathbf{E}$ as shown in Figure 4.18. From Coulomb's law, the force on Q is $\mathbf{F} = Q\mathbf{E}$ so that the *work done* in displacing the charge by $d\mathbf{l}$ is

$$dW = -\mathbf{F} \cdot d\mathbf{l} = -Q\mathbf{E} \cdot d\mathbf{l} \tag{4.58}$$

The negative sign indicates that the work is being done by an external agent. Thus the total work done, or the potential energy required, in moving Q from A to B is

$$\boxed{W = -Q \int_A^B \mathbf{E} \cdot d\mathbf{l}} \tag{4.59}$$

Dividing W by Q in eq. (4.59) gives the potential energy per unit charge. This quantity, denoted by V_{AB}, is known as the *potential difference* between points A and B. Thus

$$\boxed{V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E} \cdot d\mathbf{l}} \tag{4.60}$$

The **potential** at any point is the potential difference between that point and a chosen point at which the potential is zero.

$$V = -\int_{\infty}^{r} \mathbf{E} \cdot d\mathbf{l}$$

RELATIONSHIP BETWEEN E AND V—

As shown in the previous section, the potential difference between points A and B is independent of the path taken. Hence,

$$V_{BA} = -V_{AB}$$

that is, $V_{BA} + V_{AB} = \oint \mathbf{E} \cdot d\mathbf{l} = 0$

or

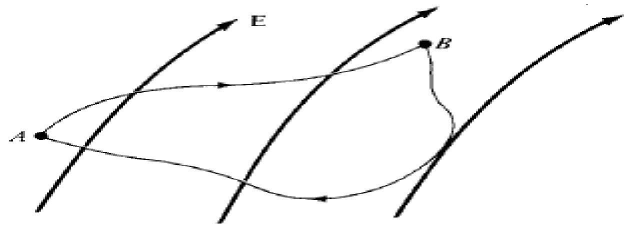
$$\boxed{\oint \mathbf{E} \cdot d\mathbf{l} = 0} \tag{4.73}$$

This shows that the line integral of $\mathbf{E}$ along a closed path as shown in Figure 4.19 must be zero. Physically, this implies that no net work is done in moving a charge along a closed path in an electrostatic field. Applying Stokes's theorem to eq. (4.73) gives

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = 0$$

or

$$\boxed{\nabla \times \mathbf{E} = 0} \tag{4.74}$$



$$dV = -\mathbf{E} \cdot d\mathbf{l} = -E_x\,dx - E_y\,dy - E_z\,dz$$

But

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

Comparing the two expressions for dV, we obtain

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}$$

Thus:

$$\boxed{\mathbf{E} = -\nabla V}$$